Graphs and Derivatives

Example: Let $f(x) = 2x^2 - 3x$

Find $f'(x)$. $= 4x - 3$

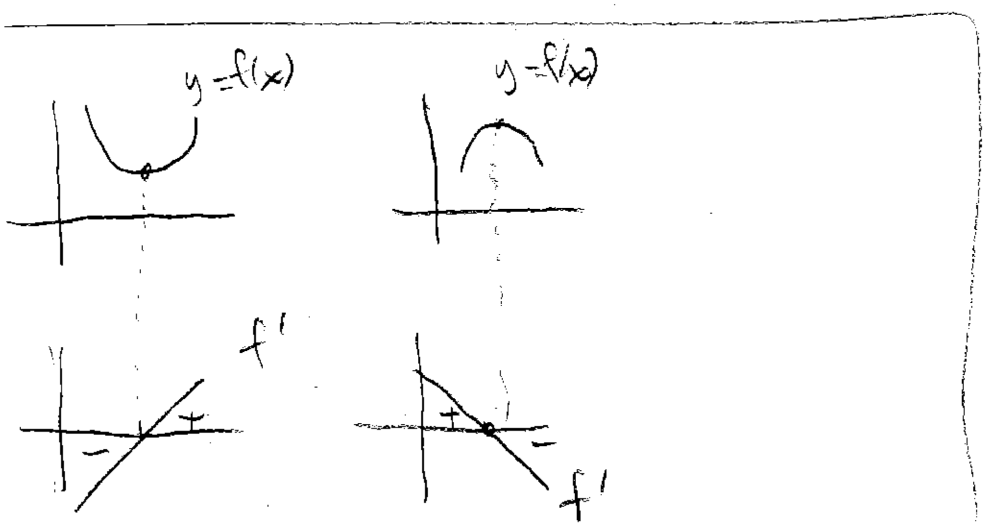
Note:

$\boxed{\text{I}}$ $f'(x) = 0$ WHEN

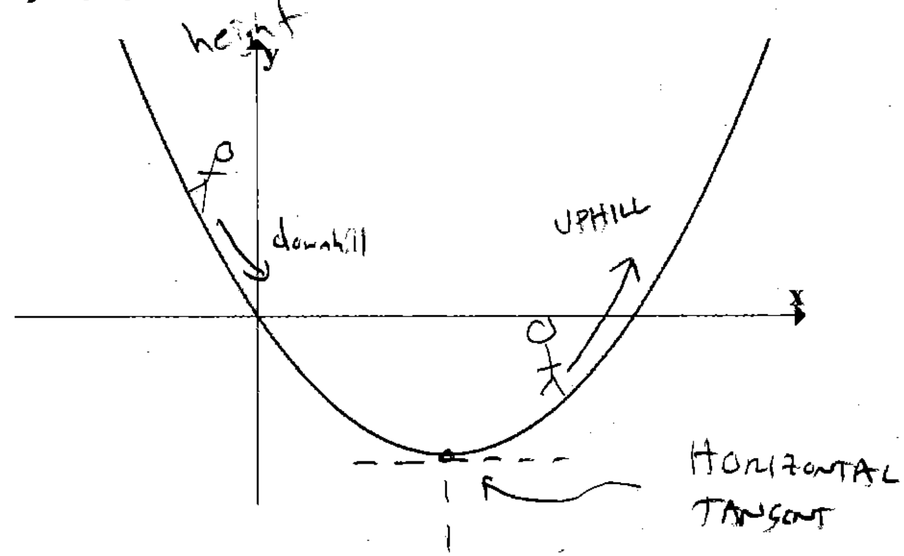
$4x - 3 = 0 \iff x = 3/4$

$\boxed{\text{II}}$ f' IS <u>POSITIVE</u> FOR ALL $x > 3/4$

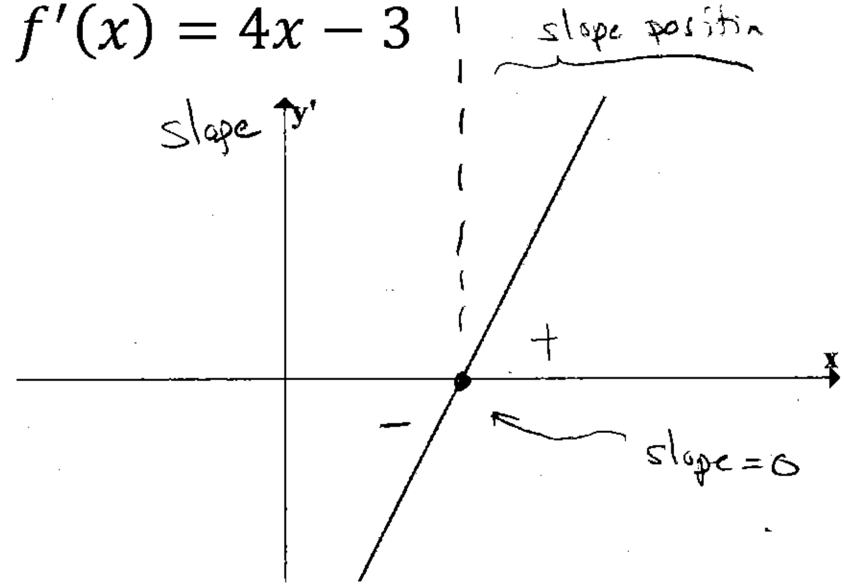
$\boxed{\text{III}}$ f' IS <u>NEGATIVE</u> FOR ALL $x < 3/4$



$f(x) = 2x^2 - 3x$



$f'(x) = 4x - 3$



Notes/Observations: Given $y = f(x)$.

- $y = f'(x)$ is a new function.
- $f(x) =$ "height of the graph at *x*"
- $f'(x) =$ "slope of $f(x)$ at x"
- $f'(x)$ is "instantaneous rate of change" (speedometer speed)
- The units of $f'(x)$ are $\frac{\text{y-units}}{\text{x-units}}$.

Fundamental to all applications:

$f(x)$	$f'(x)$
horiz. tangent	zero
increasing	positive
decreasing	negative

Old Exam Question:

The height of a balloon after t seconds is given by

$$B(t) = 15t^2 - t^3 \qquad \text{feet.}$$

a. At time t = 1 second, is the balloon rising or falling?

b. Find the maximum height reached by the balloon.

(a) $B'(t) = 30t - 3t^2$

$B'(1) = 30(1) - 3(1)^2 = 27$

POSITIVE!

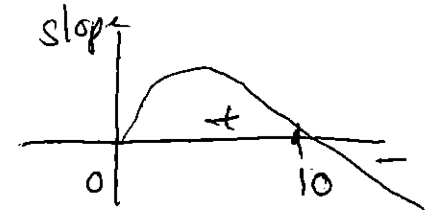
RISING

(b) $B'(t) = 30t - 3t^2 \stackrel{?}{=} 0$

$10t - t^2 = 0$

$t(10 - t) = 0$

$t = 0 \quad \text{or} \quad t = 10$



FROM $t=0$ to $t=10$, B RISING.
AFTER $t=10$, B FALLING.

MAX HEIGHT OCCURS AT $t=10$

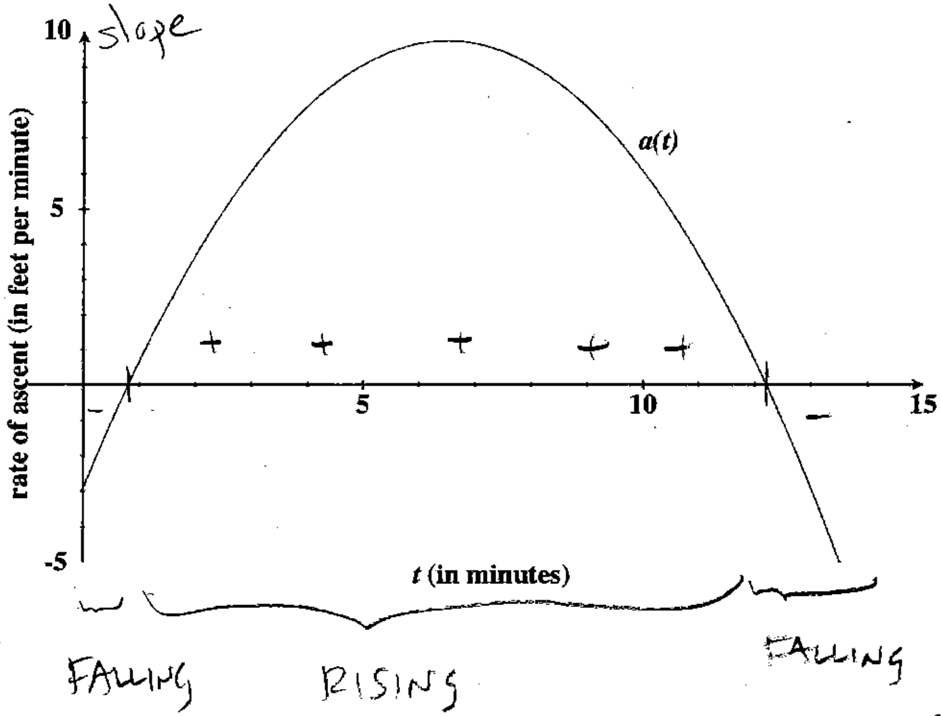
$B(10) = 15(10)^2 - (10)^3$

$= \boxed{500 \text{ feet}}$

9.9 HW Problem 8:

Rate of ascent for a balloon (in feet per minute) is given by

$$a(t) = -0.3t^2 + 3.9t - 2.928$$



How will you answer these:

(a) Find the longest integral over which Balloon A is rising.

(d) Find the time at which the balloon is rising the fastest?

(a) NEED TO FIND WHEN $a(t)$ IS POSITIVE

$\boxed{\text{STEP 1}}$ FIND WHERE $a(t) \stackrel{?}{=} 0$.

$-0.3t^2 + 3.9t - 2.928 \stackrel{?}{=} 0$

QUAD. FORMULA

$\boxed{\text{STEP 2}}$ INTERPRET BETWEEN THESE

(d) WANT TO FIND HIGHEST POINT ON $a(t)$ (HORIZ. TANGENT)

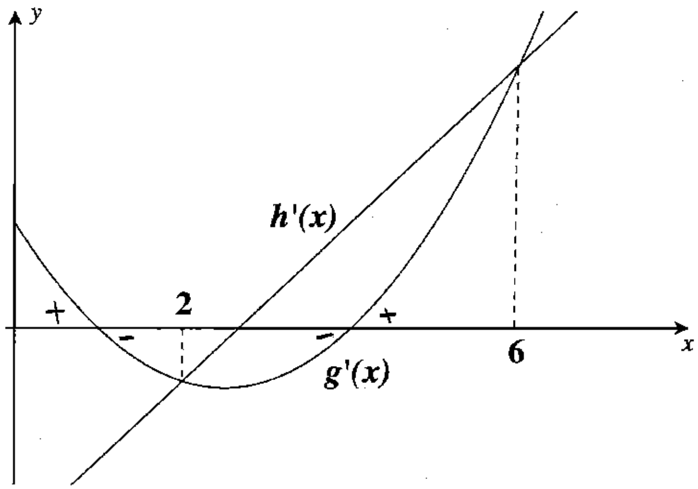
$\boxed{\text{STEP 1}}$ $a'(t) = -0.6t + 3.9$

$\boxed{\text{STEP 2}}$ Solve $a'(t) \stackrel{?}{=} 0$

HW 9.9/1:

Given $g'(x) = 2x^2 - 10x + 8$

$h'(x) = 6x - 16$



What does it mean when…

(a) … g'(x) crosses the x-axis?

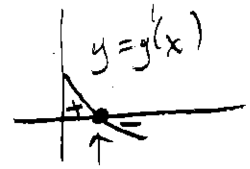
… h'(x) crosses the x-axis?

(b) … g'(x) has a horizontal tangent (and how do you find it)?

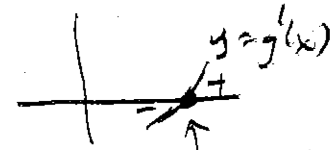
(c) … h'(x) intersects g'(x)?

(a)

$g'(x) \stackrel{?}{=} 0 \iff g(x)$ has a horizontal tangent



CORRESPONDS TO A MAX



CORRESPONDS TO A MIN.

$h'(x) = 0 \iff h(x)$ has a horiz. tangent

CORRESPONDS TO A MIN.

(b) g' has a horiz. tangent $\iff g''(x) \stackrel{?}{=} 0$

$4x - 10 \stackrel{?}{=} 0$

(c) $g'(x) = h'(x) \iff$ SAME SLOPE (RATE)

$\iff$ g(x) AND h(x) BIGGEST GAPS